Mass and Decay Constant of $I = 1/2$ Scalar Meson In QCD Sum Rule

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Abstract

We calculate the mass and decay constant of $I = \frac{1}{2}$ scalar mesons composed of quark-antiquark pairs based on QCD sum rule. The quauk-antiquark pairs can be $s\bar{q}$ or $q\bar{s}$ $(q = u, d)$ in quark model, the quantum numbers of spin and orbital angular momentum are $S = 1$, $L = 1$. We obtain the mass of the ground sate in this channel is 1.410 ± 0.049GeV. This result favors that $K_0^*(1430)$ is the lowest scalar state of $s\bar{q}$ or $q\bar{s}$. We also predict the first excited scalar resonance of $s\bar{q}$ is larger than 2.0 GeV.

1. Introduction

Glueball and scalar mesons should exist according to QCD and quark model. Some scalar mesons below 2 GeV have been observed, such as, i) for isospin $I = 0, 1$ states: $f_0(600)$ or σ, $a_0(980)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$; ii) for $I = \frac{1}{2}$ states: $\kappa(900)$ and $K_0^*(1430)$ [1, 2, 3, 4]. The number of these scalar mesons exceeds the particle states which can be accommodated in one nonet in the quark model. It is believed that there are two nonets below and above 1 GeV [5, 6]. The components of the meson states in each nonet have not been completely determined yet. For the scalar mesons below 1 GeV there are several interpretations. They are interpreted as meson-meson molecular states [7] or multi-quark states $qq\bar{q}\bar{q}$ [8], etc.. However, from the theoretical

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point of view there must be quark-antiquark SU(3) scalar nonet. Therefore it is important to determine the masses of the ground states of $q\bar{q}$ with quantum number $J^P = 0^+$ based on QCD. For isospin $I = 0, 1$ states different quark flavor may mix, and scalar $q\bar{q}$ states may also mix with scalar glueball if they have the same quantum number of J^{PC} and similar masses [9]-[14]. Some authors have tried to determine the mixing angles of the glueball with $q\bar{q}$ scalar mesons by using decay patterns of some scalar mesons [14]-[17]. These works imply that glueball possibly mix with $q\bar{q}$ scalar mesons. For $I = 1/2$ states, they cannot mix with glueball because they have strange quantum number. The physical state is directly the $s\bar{q}$ and $q\bar{s}$ bound state. Therefore the mass of the ground state of $s\bar{q}$ or $q\bar{s}$ can be determined without necessity for considering mixing effect.

QCD sum rule is a powerful tool to calculate hadronic nonpertubative parameters based on QCD [18]. It has been used to calculate the masses and decay constants of $0^{-+}, 1^{-+}, 2^{++}$ mesons before and give satisfactory results [18, 19, 20, 21].

In this paper, we calculate the mass and decay constant of $I = 1/2$ scalar meson with QCD sum rule. We find that it is impossible to obtain $s\bar{q}$ scalar meson mass below 1 GeV from QCD sum rule. The most favorable result for the mass of $s\bar{q}$ scalar meson is 1.410 ± 0.049GeV. Therefore, if $\kappa(900)$ is $s\bar{q}$ scalar bound state, this would be a big problem for QCD. This problem can be solved by assuming that $\kappa(900)$ is irrelevant to $s\bar{q}$ scalar channel, $< 0|\bar{s}q|\kappa(900) >\sim 0$, and $K_0^*(1430)$ is the scalar ground state of $s\bar{q}$ or $q\bar{s}$. With this assumption, calculation based on QCD will be consistent with experiment. Therefore, our result favors that $K_0^*(1430)$ is the lowest scalar bound state of $s\bar{q}$.

If this is correct, then from the approximate SU(3) flavor symmetry, the masses of the other $J^P = 0^+$ mesons in the scalar nonet should be slightly above or below 1.4 GeV. This result would imply that scalars with masses below 1 GeV are not dominated by quark-antiquark pairs. This is consistent with the calculation of lattice QCD which implies that a nonet of quark-qntiquark scalars is in the range 1.2-1.6 GeV [22].

The remaining part of this paper is organized as follows. In Section 2, we briefly introduce the process to calculate the scalar meson with QCD sum rule and get the Wilson coefficients for the corresponding two-point scalar current correlation function. Section 3 is devoted to numerical analysis and conclusion.

2. The method

To calculate the mass of scalar $s\bar{q}$ or $q\bar{s}$ meson, the two-point correlation function should be taken as

$$\Pi(q^2) = i\int d^4xe^{iq\cdot x}\langle 0|T\{j(x)j^+(0)\}|0\rangle \tag{1}$$

where $j(x) = \bar{s}(x)q(x)$, $j^+(0) = \bar{q}(0)s(0)$.

On one hand, the correlation function can be expressed based on the dispersion relation in terms of hadron states

$$\Pi^h(q^2) = \frac{1}{\pi}\int \frac{ds\hat{I}_m\Pi(s)}{s-q^2} \tag{2}$$

where $\hat{I}_m\Pi(s)$ is the imaginary part of the two-point correlation function, which can be obtained by inserting a complete set of quantum states $\sum|n\rangle\langle n|$ into Eq.(1). The result is

$$2\hat{I}_m\Pi(s) = \sum_n 2\pi\delta(s-m_n^2)\langle 0|j(0)|n\rangle\langle n|j^+(0)|0\rangle \tag{3}$$

For the scalar states S, its decay constant f_S can be defined through

$$\langle 0|j(0)|S\rangle = m_S f_S \tag{4}$$

where m_S is the mass of the scalar state. Based on Eq.(2)- Eq.(4), and explicitly separating out the lowest scalar state, the correlation function can be expressed as

$$\Pi^h(q^2) = \frac{m_S^2f_S^2}{m_S^2-q^2} + \frac{1}{\pi}\int_{s^0}^{\infty}\frac{ds\rho^h(s)}{s-q^2} \tag{5}$$

where $\rho^h(s)$ expresses the contribution of higher resonances and continuum state, s^0 is the threshold of higher resonances and continuum state.

On the other hand, the correlation function can be expanded in terms of operator-product expansion at large negative value of q^2.

$$\begin{aligned}\Pi^{QCD}(q^2) &= i\int d^4xe^{iq\cdot x}\langle 0|T\{j(x)j^+(0)\}|0\rangle \\ &= C_0I + C_3\langle 0|\bar{\Psi}\Psi|0\rangle + C_4\langle 0|G^a_{\alpha\beta}G^{a\alpha\beta}|0\rangle + C_5\langle 0|\bar{\Psi}\sigma_{\alpha\beta}T^aG^{a\alpha\beta}\Psi|0\rangle \\ &+ C_6\langle 0|\bar{\Psi}\Gamma\Psi\bar{\Psi}\Gamma'\Psi|0\rangle + \cdots \end{aligned} \tag{6}$$

where C_i, $i = 0, 3, 4, 5, 6, \cdots$ are Wilson coefficients, I is the unit operator, $\bar{\Psi}\Psi$ is the local Fermion field operator of light quarks, $G^a_{\alpha\beta}$ is gluon strength

tensor, Γ and Γ' are the matrices appearing in the procedure of calculating the Wilson coefficients.

For convenience later, we reexpress the above equation as

$$\Pi^{QCD}(q^2) = \frac{1}{\pi}\int \frac{ds\rho^{pert}}{s-q^2} + \rho_3^{nonp} + \rho_4^{nonp} + \rho_5^{nonp} + \rho_6^{nonp} + \cdots \tag{7}$$

where $\rho_3^{nonp}, \cdots, \rho_6^{nonp}, \cdots$ are contributions of condensates of dimension 3, 4, 5, 6,$\cdots$ in Eq.(6).

Matching $\Pi^h(q^2)$ with $\Pi^{QCD}(q^2)$ we can get the equation which relates mass of scalar meson with QCD parameters and a few condensate parameters. In order to suppress the contribution of higher resonances and that of condensate terms, we make Borel transformation over q^2 in both sides of the equation, the Borel transformation is defined as

$$\hat{B}_{|\ p^2,M^2} f(q^2) = \lim_{\substack{n\to\infty \\ q^2\to-\infty \\ -q^2/n = M^2}} \frac{(-q^2)^n}{(n-1)!}\frac{\partial^n}{\partial(q^2)^n} f(q^2).$$

After assuming quark-hadron duality, i.e., by assuming that the contribution of higher resonance and continuum states can be approximately cancelled by the perturbative integration over the threshold s^0 [23], the resulted sum rules for the mass and decay constant of the scalar meson are

$$m_S = \sqrt{\frac{R_1}{R_2}} \tag{8}$$

$$f_S = \frac{1}{m_S}\sqrt{e^{m_S^2/M^2} R_2} \tag{9}$$

where

$$R_1 = \frac{1}{\pi}\int_{(m_1+m_2)^2}^{s^0} ds s\rho^{pert}(s)e^{-s/M^2} + M^4[\frac{\partial(M^2\hat{B}\rho_3^{nonp})}{\partial M^2}] + M^4[\frac{\partial(M^2\hat{B}\rho_4^{nonp})}{\partial M^2}] + M^4[\frac{\partial(M^2\hat{B}\rho_5^{nonp})}{\partial M^2}] + M^4[\frac{\partial(M^2\hat{B}\rho_6^{nonp})}{\partial M^2}] \tag{10}$$

$$R_2 = \frac{1}{\pi}\int_{(m_1+m_2)^2}^{s^0} ds\rho^{pert}(s)e^{-s/M^2} + M^2\hat{B}\rho_3^{nonp} + M^2\hat{B}\rho_4^{nonp} + M^2\hat{B}\rho_5^{nonp} + M^2\hat{B}\rho_6^{nonp} \tag{11}$$

where $\hat{B}\rho_i^{nonp}$ express Borel transformation of ρ_i^{nonp}, M is Borel parameter, and m_1 and m_2 are the masses of the two light quarks.

We need to calculate the Wilson coefficients in Eq.(6) to get the mass and decay constant of scalar meson. Collecting the contribution of diagrams in Fig.1, we get the result of $\hat{B}\rho_i^{nonp}$ which is listed in Appendix.

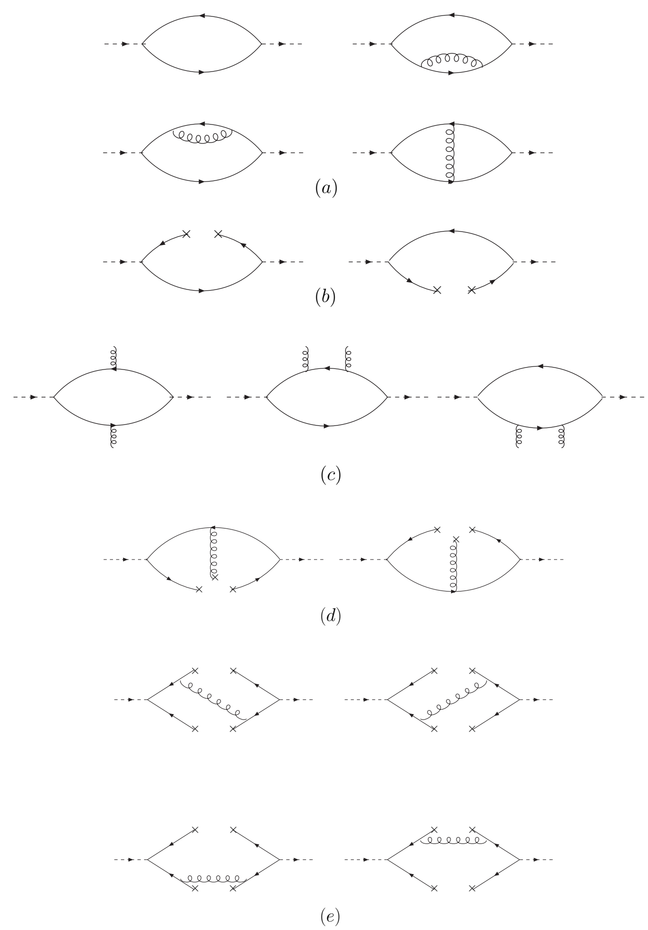

Figure 1: Diagrams for the contribution to Wilson coefficients. (a): diagrams contribute to unit operator; (b): diagrams contribute to bi-quark operators $\bar{\Psi}(x)\Psi(0)$; (c): diagrams contribute to $G^a_{\mu\nu}G^{a\mu\nu}$; (d): diagrams contribute to quark-gluon mixing $\bar{\Psi}(x)\Psi(0)G^a_{\mu\nu}$; (e): diagrams contribute to four-quark operators $\langle\bar{\Psi}\Psi\rangle^2$

3. Numerical analysis and conclusion

The numerical parameters used in this paper are taken as [18, 21]

$$\langle\bar{q}q\rangle = -(0.24 \pm 0.01\text{GeV})^3, \qquad \langle\bar{s}s\rangle = m_0^2\langle\bar{q}q\rangle$$

$$\alpha_s\langle GG\rangle = 0.038\text{GeV}^4, \qquad g\langle\bar{\Psi}\sigma T\Psi\rangle = m_0^2\langle\bar{\Psi}\Psi\rangle$$

$$\alpha_s\langle\bar{\Psi}\Psi\rangle^2 = 6.0\times 10^{-5}\text{GeV}^6, m_0^2 = 0.8\pm 0.2\text{GeV}^2$$
$$m_s = 0.14\text{GeV}, \qquad m_u \approx m_d = 0.005\text{GeV} \tag{12}$$

For the choice of Borel parameter M^2, as in [18, 24], we define $f_{thcorr}(M^2)$ as $m(M^2)$ in Eq.(8) without the continuum contribution ($s^0 = \infty$) and $m_{nopower}(M^2)$ as $m(M^2)$ in Eq.(8) without power corrections, then define $f_{nopower}(M^2)$ as $m(M^2)/m_{nopower}(M^2)$ and f_{cont} as $m(M^2)/f_{thcorr}(M^2)$. To get reliable prediction of the mass in QCD sum rule, f_{cont} should be limited to above 90% to suppress the contribution of higher resonance and continuum, and $f_{nopower}(M^2)$ be limited to less than 10% deviation from 1, which can ensure condensate contribution much less than perturbative contribution.

There are two low mass scalar meson states with isospin $I = 1/2$ and strange number $|S| = 1$ found in experiment. They are $\kappa(900)$ with mass m_κ about $800 \sim 900$MeV [2, 3, 4], and $K_0^*(1430)$ with mass $m(K_0^*(1430)) = 1.412\pm 0.006$GeV [1]. In theory, taking appropriate value for the threshold parameter s^0, one can separate out the contribution of the lowest resonance in QCD sum rule. We vary the value of the threshold parameter s^0, and find that it is impossible to obtain the mass of $\kappa(900)$ with the sum rule in Eq.(8). There is no stable 'window' for the Borel parameter in this mass region. Therefore, if $\kappa(900)$ is the lowest scalar state in the $s\bar{q}$ channel, it would be a big problem for QCD sum rule. However, if we increase the value of s^0, i.e., for $s^0 = 4.0 \sim 4.8$ GeV^2, we does find the stable 'window' for Borel parameter, which is shown in Fig.2. The resulted stable window is in the range $1.0 < M^2 < 1.2$ GeV^2.

Fig.2(a) shows that between the arrows A and B, both the contributions of condensate and higher resonance are less than 10%. So in this region, the operator product expansion is effective, and the assumption of quark-hadron duality does not seriously affect the numerical result, which means that QCD sum rule can give reliable prediction in this parameter space. For $s^0 = 4.0 \sim 4.8$ GeV^2, the mass of scalar $s\bar{q}$ meson in QCD sum rule is

$$m(s\bar{q}) = 1.410\pm 0.049 \text{ GeV} \tag{13}$$

where the error bar is estimated by the variation of Borel parameter in the range $1.0 < M^2 < 1.2$ GeV^2, the variation of s^0 within $4.0 \sim 4.8$ GeV^2,

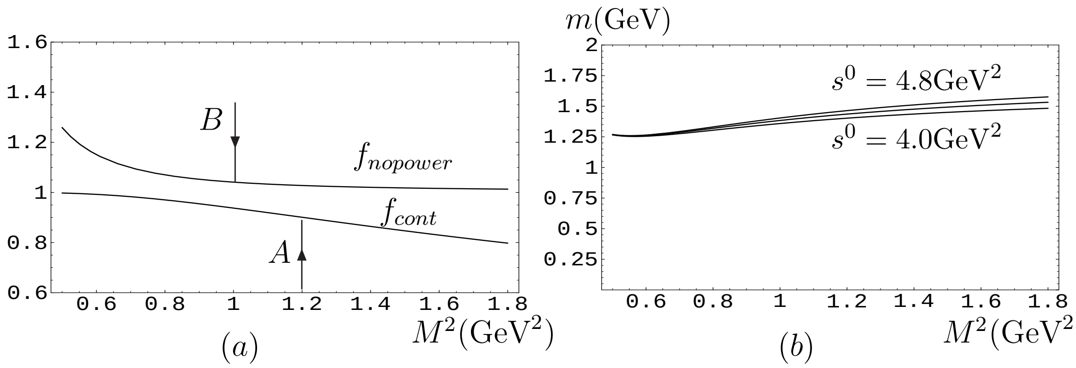

Figure 2: (a) The region between the arrow A and B is reliable for determining the mass ($s^0 = 4.4GeV^2$). (b) The curves correspond to the mass of scalar $s\bar{d}$ meson for the continuum threshold $s^0 = 4.0GeV^2, s^0 = 4.4GeV^2, 4.8GeV^2$, respectively. The central one is for $s^0 = 4.4GeV^2$.

the uncertainty of higher α_s correction for the perturbative diagram and the condensate parameters. The variation of Borel parameter yields $\pm 1.8\%$ uncertainty for the mass, s^0 yields $\pm 2.0\%$, α_s correction gives $\pm 2.2\%$, the uncertainty caused by the condensate parameters is less than 0.6%. All the uncertainties are added quadratically.

The energy scale for the $\alpha_s(\mu)$ correction is taken to be $\mu = M$. In the stable window, the range of Borel parameter is $1.0 < M^2 < 1.2$GeV^2, therefore $\alpha_s(M) \sim 0.5$. We checked that the contribution of the α_s correction at first order is about 2.2%, which is not large. This can be understood because most contribution of the α_s correction is cancelled between the numerator and denominator of Eq.(8). We use 2.2% to estimate the uncertainty caused by the higher order α_s corrections.

On one hand, it is impossible to obtain the mass of lower scalar state $\kappa(900)$ from QCD sum rule for $s\bar{q}$ channel. If regard $\kappa(900)$ as $s\bar{q}$ scalar bound state, it would be a big problem for QCD. On the other hand, QCD sum rule can give most favorable mass which is consistent with the mass of $K_0^*(1430)$. Therefore it is acceptable to assume that $\kappa(900)$ is irrelevant to $s\bar{q}$ scalar bound state, and

$$< 0|\bar{s}q|\kappa(900) > \sim 0 \qquad (14)$$

With this assumption, $K_0^*(1430)$ can be accepted as the lowest scalar bound

state of $s\bar{q}$. Then there will be no problem between QCD and experiment.

One may still be afraid that there are contributions of the lower mass state $\kappa(900)$ mixed in the result of eq.(13) in fact. If this is indeed the case, the result of the sum rule may be some weighted average of the two resonances of $\kappa(900)$ and $K_0^*(1430)$. Therefore this situation should be carefully checked. Because the sum rule for the mass of the scalar bound state in eqs.(8), (10) and (11) includes the spectrum integration $\int_{(m_1+m_2)^2}^{s^0} ds$, in principle one can lower the value of s^0 to separate the lowest bound state. Therefore, we checked what result for the mass can be got by lower the value of s^0 within the stable window $1.0 < M^2 < 1.2\mathrm{GeV}^2$ selected in Fig.2(a). The result is shown in Fig.3. It shows that for any value of s^0, the possible mass is large than 960MeV,

$$m(s\bar{q}) > 960 \text{ MeV} \tag{15}$$

Therefore the possible effect of $\kappa(900)$ can be safely ruled out in the sum rule result in eq.(13). Note that the most recent experimental result for the mass of $\kappa(900)$ from E791 collaboration is $m_\kappa = 797 \pm 19 \pm 42\mathrm{MeV}$ [3].

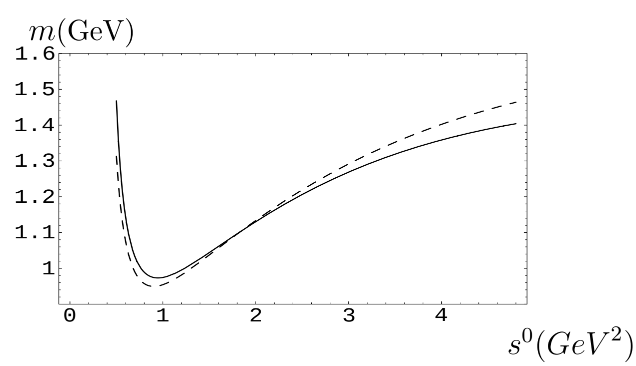

Figure 3: The possible mass result by varying the value of the threshold s^0. The solid curve is for the Borel parameter $M^2 = 1.0\mathrm{GeV}^2$, and the dashed one for $M^2 = 1.2\mathrm{GeV}^2$.

If $K_0^*(1430)$ is the ground state of $s\bar{q}$ or $q\bar{s}$, from the approximate SU(3) flavor symmetry, the masses of the other $J^P = 0^+$ mesons in the scalar nonet should be also around 1.4 GeV. This implies that the scalars with masses less than 1 GeV, i.e., $f_0(600)$, $a_0(980)$, $f_0(980)$ etc., can not be dominated by quark-antiquark bound states. This is consistent with the calculation of lattice QCD which implies that a nonet of quark-antiquark scalars is in the region 1.2-1.6 GeV [22].

Our result can be further checked by experiment. From the threshold parameter s^0, we can predict that the mass of the first excited resonance in $s\bar{q}$ scalar channel should be larger than $\sqrt{s^0}$, that is

$$m^*(K_0^*) > 2.0 \text{ GeV} \tag{16}$$

This prediction can be tested by experiment.

Next we discuss the decay constant of the two-quark scalar bound state $s\bar{q}$. From the above analysis, we take the threshold parameter $s^0 = 4.0 \sim 4.8$ GeV^2. Consider $K_0^*(1430)$ as the only resonance below 2 GeV in the $s\bar{q}$ scalar channel, we can obtain the decay constant of $K_0^*(1430)$ as a function of Borel parameter M^2 (see eq.(9)). The numerical result is shown in Fig.4.

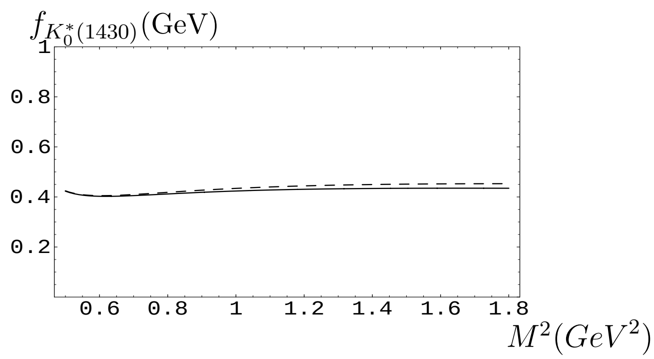

Figure 4: The decay constant of $K_0^*(1430)$ as a function of the Borel parameter M^2. The solid curve is for $s^0 = 4.0\text{GeV}^2$, and the dashed one for $s^0 = 4.8\text{GeV}^2$.

Fig.4 shows that the decay constant is very stable. The determined stable 'window' is still in $1.0 < M^2 < 1.2\text{GeV}^2$, where the continuum and condensate contribution are restricted to be less than 15% and 4%, respectively. Within this stable window, the decay constant of $K_0^*(1430)$ is

$$f(K_0^*(1430)) = 427 \pm 85 \text{ MeV} \tag{17}$$

The variation of s^0 yields $\pm 30\%$ uncertainty for the decay constant, α_s correction gives $\pm 20\%$, the uncertainties caused by the condensate parameters and the variation of Borel parameter are less than 0.3% and 0.1%, respectively. All the uncertainties are added quadratically to give the error bar in the above result.

Again we should check what will happen if we consider two resonances $\kappa(900)$ and $K_0^*(1430)$ existing below 2 GeV in our sum rule analysis. Therefore

we add one more resonance into eq.(5), then matching $\Pi^h(q^2)$ with $\Pi^{QCD}(q^2)$ in eq.(7). By assuming quark-hadron duality to cancel the contribution of higher resonance and continuum above 2 GeV, and making Borel transformation in both sides, we get the Borel improved matching equation

$$m_{S1}^2 f_{S1}^2 e^{-m_{S1}^2/M^2} + m_{S2}^2 f_{S2}^2 e^{-m_{S2}^2/M^2} = R_2 \tag{18}$$

where R_2 has been given in eq.(11), and m_{S1}, m_{S2} are fixed to be the masses of $\kappa(900)$ and $K_0^*(1430)$, $m_{S1} = 900$MeV, $m_{S2} = 1410$MeV. f_{S1} and f_{S2} are the decay constants of the relevant scalar mesons.

Differentiate both sides of eq.(18) with the operator d/dM^2, we can get another equation

$$m_{S1}^4 f_{S1}^2 e^{-m_{S1}^2/M^2} + m_{S2}^4 f_{S2}^2 e^{-m_{S2}^2/M^2} = R_1 \tag{19}$$

where R_1 is defined in eq.(10). With eqs.(18) and (19), we can obtain

$$f_{S1}^2 = \frac{e^{m_{S1}^2/M^2}}{m_{S1}^2(m_{S2}^2 - m_{S1}^2)}(m_{S2}^2 R_2 - R_1) \tag{20}$$

$$f_{S2}^2 = \frac{e^{m_{S2}^2/M^2}}{m_{S2}^2(m_{S1}^2 - m_{S2}^2)}(m_{S1}^2 R_2 - R_1) \tag{21}$$

From the above result we can perform the numerical analysis for the decay constants in the two-resonance ansatz. The numerical result is shown in Fig.5.

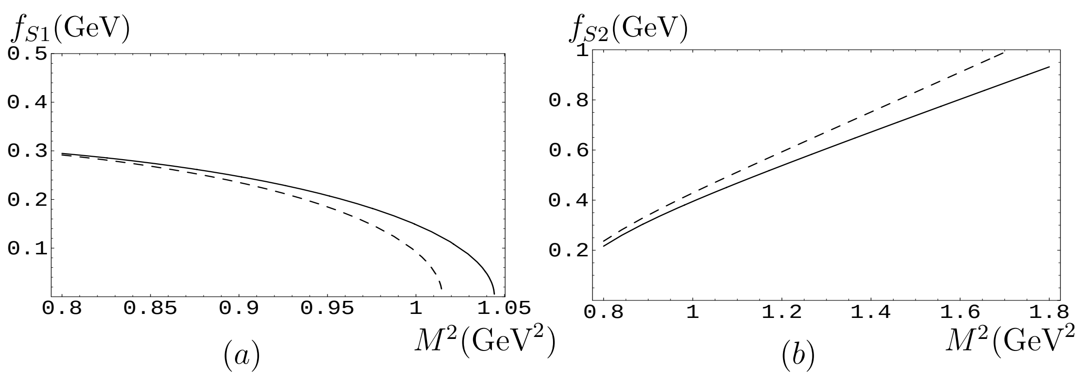

Figure 5: The decay constants in two-resonance ansatz below 2GeV . The solid curve is for $s^0 = 4.0$GeV^2, and the dashed one for $s^0 = 4.8$GeV^2. (a) The decay constant of the low resonance $\kappa(900)$. (b) The decay constant of the higher resonance $K_0^*(1430)$

From Fig.5, we can see that both the two decay constants are unstable as a function of Borel parameter in the two-resonance ansatz. Adding the

lower resonance $\kappa(900)$ in the sum rule analysis for the $s\bar{q}$ channel spoils the stability existing in the one-resonance ansatz, which is shown in Fig.4. From the requirement of numerical stability of QCD sum rule, the numerical analysis of the decay constant does not favor to include $\kappa(900)$ in $s\bar{q}$ scalar channel. In addition, we can see from Fig.5a that the decay constant of the lower scalar resonance $\kappa(900)$ tend to be zero at $M^2 \sim 1.01$ and 1.05 GeV. This is consistent with the requirement that $< 0|\bar{s}q|\kappa(900) >\sim 0$ in the one-resonance ansatz, where the stability window is located in the range $1.0 < M^2 < 1.2$ GeV.

Therefore, both the analyses of the mass and decay constant of $s\bar{q}$ scalar meson from QCD sum rule imply that $\kappa(900)$ is not dominated by quark-antiquark bound state, and the lowest $s\bar{q}$ scalar bound state is $K_0^*(1430)$. The mass obtained from QCD sum rule is

$$m(K_0^*(1430)) = 1.410 \pm 0.049\text{GeV} \tag{22}$$

and the decay constant is

$$f(K_0^*(1430)) = 427 \pm 85\text{MeV}. \tag{23}$$

In summary, we calculate the mass and decay constant of scalar meson $s\bar{q}$ in QCD sum rule. Our result favors that $K_0^*(1430)$ is the ground state of $s\bar{q}$ scalar bound state. If this is correct, it would imply that scalar mesons below 1 GeV are not dominated by quark-antiquark pairs. We also predict that the mass of the first excited resonance of $s\bar{q}$ scalar bound state is larger than 2.0 GeV.

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Appendix

Borel transformed coefficients of perturbative and nonperturbative contributions $\hat{B}\rho_i^{nonp}$ in Eqs.(10) and (11) are listed below

$$\rho^{pert}(s) \quad = \quad \{\frac{-3[(m_1+m_2)^2 - s]\sqrt{(-(m_1-m_2)^2+s)(-(m_1+m_2)^2+s)}}{8\pi s}$$

$$+\frac{3s}{8\pi}\frac{13}{3}\frac{\alpha_s(\mu)}{\pi}\}e^{-s/M^2} \tag{24}$$

where the term with $\alpha_s(\mu)$ is the radiative correction to the perturbative contribution [20], and the scale is taken to be $\mu = M$.

$$\begin{aligned}\hat{B}\rho_3^{nonp} &= [3M^4m_1m_2^2 + 3M^2m_1^2m_2^3 + m_1^3m_2^4 + 3M^6(m_1 + 2m_2)] \\ &\quad \langle\bar{s}s\rangle\frac{e^{-m_2^2/M^2}}{6M^8} + [3M^4m_1^2m_2 + 3M^2m_1^3m_2^2 + m_1^4m_2^3 \\ &\quad +3M^6(2m_1 + m_2)]\langle\bar{d}d\rangle\frac{e^{-m_1^2/M^2}}{6M^8}\end{aligned} \tag{25}$$

$$\begin{aligned}\hat{B}\rho_4^{nonp} &= 4\pi\alpha_s\langle GG\rangle\{\frac{-3(m_1+m_2)^2}{256e^{((m_1+m_2)^2/M^2)}M^2m_1m_2\pi^2} \\ &\quad +(3M^4m_1^2m_2 + 3M^2m_1^3m_2^2 + m_1^4m_2^3 + 3M^6(2m_1 + m_2)) \\ &\quad \frac{1}{288e^{(m_1^2/M^2)}M^8m_2\pi^2} \\ &\quad +(3M^4m_1m_2^2 + 3M^2m_1^2m_2^3 + m_1^3m_2^4 + 3M^6(m_1 + 2m_2)) \\ &\quad \frac{1}{288e^{(m_2^2/M^2)}M^8m_1\pi^2} \\ &\quad +\frac{-12m_1(m_1-m_2)^2m_2 + M^2(-7m_1^2 + 26m_1m_2 - 7m_2^2)}{768e^{((m_1-m_2)^2/M^2)}M^4m_1m_2\pi^2} \\ &\quad \int_{(m_1+m_2)^2}^{\infty}dt\{\frac{3(m_1+m_2)^4}{128e^{[(m_1+m_2)^2/M^2]}M^4\pi^2(m_1^2 + 2m_1m_2 + m_2^2 - t)} \\ &\quad +\frac{m_1m_2(m_1^2 - m_1m_2 + m_2^2 - t)t^2}{8e^{(t/M^2)}M^4\pi^2(m_1^2 - 2m_1m_2 + m_2^2 - t)^2(m_1^2 + 2m_1m_2 + m_2^2 - t)} \\ &\quad -\{(m_1-m_2)^2[4m_1(m_1-m_2)^2m_2(m_1^2 - 2m_1m_2 + m_2^2 - t) \\ &\quad +M^2(3m_1^4 - 16m_1^3m_2 + 26m_1^2m_2^2 - 16m_1m_2^3 + 3m_2^4 - \\ &\quad 3m_1^2t + 14m_1m_2t - 3m_2^2t)]\} \\ &\quad \frac{1}{128e^{((m_1-m_2)^2/M^2)}M^6\pi^2(m_1^2 - 2m_1m_2 + m_2^2 - t)^2}\} \\ &\quad \frac{1}{\sqrt{[(-(m_1-m_2)^2 + t)(-(m_1+m_2)^2 + t)]}}\}\end{aligned} \tag{26}$$

$$\begin{aligned}\hat{B}\rho_5^{nonp} &= g\langle\bar{\Psi}\sigma T\Psi\rangle\{-\frac{m_1[-6M^4 + m_1^3m_2 + 3M^2m_1(m_1+m_2)]}{12e^{(m_1^2/M^2)}M^8} \\ &\quad -\frac{m_2[-6M^4 + m_1m_2^3 + 3M^2m_2(m_1+m_2)]}{12e^{(m_2^2/M^2)}M^8}\}\end{aligned} \tag{27}$$

$$
\begin{aligned}
\hat{B}\rho_6^{nonp} &= 4\pi\alpha_s\langle\bar{\Psi}\Psi\rangle^2\{\frac{4(m_1+m_2)^2}{9M^2m_1^2m_2^2}+[-(m_1^2m_2^6)+m_2^8 \\
&+36M^6m_1(m_1+2m_2)+84M^4m_2^2(m_1^2-m_2^2) \\
&+15M^2m_2^4(m_1^2-m_2^2)]\frac{1}{81e^{(m_2^2/M^2)}M^8m_2^2(-m_1^2+m_2^2)} \\
&+[36M^6m_2(2m_1+m_2)+m_1^6(m_1^2-m_2^2) \\
&-84M^4(m_1^4-m_1^2m_2^2)-15M^2(m_1^6-m_1^4m_2^2)] \\
&\frac{1}{81e^{(m_1^2/M^2)}M^8m_1^2(m_1^2-m_2^2)}\} \qquad (28)
\end{aligned}
$$

where $m_1 = m_s$, $m_2 = m_q$.

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